Vibration Control of Smart Cantilever Beam Using Fuzzy Logic Controller

Kamalpreet Singh, Rajeev Kumar, Mohammad Talha, and Vikas Narain

Abstract This paper presents active vibration control of smart cantilever beam using collocated piezoelectric sensor and actuator. The vibrating response of piezolaminated cantilever beam is modeled using lumped mass approach. Fuzzy logic controller is used to control the vibration, and 49 rules have been established to develop the controller. Input sensor voltage and rate of change of sensor voltage are considered as inputs while actuator voltage is considered as output. Eight combinations of different membership functions have been considered. Finally, it has been observed that Gaussian-type membership function controls the vibration fast.

Keywords Lumped parameter model · Vibration control · Piezoelectric · Fuzzy logic controller

1 Introduction

Structural vibration is one of the key issues in most of the industries such as mechanical, defense, aerospace and communication. Surface precision requirements of structures used in aforementioned industries are rigorous, and structural vibrations make them suitable for a long period of time. Active and passive vibration controls are the two strategies implemented to attenuate the structural vibrations [1]. The passive vibration control strategy involves the application of mass, spring and damper treatments to the vibrating structures [2]. In active vibration control, the host structure is sandwiched between two piezoelectric layers. One piezoelectric layer acts as a sensor while the other piezoelectric layer acts as an actuator. As the structure gets external disturbance, the sensor layer senses the disturbance and sends signal in the form of voltage to controller [3]. The controller sends the voltage signal to the actuator which after amplification is supplied to the actuator layer resulting in control force. In the last few decades, the piezoelectric ceramics of lead zirconate titanate Pb(Zr1-xTix)O_3

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(PZT) family have been extensively studied by various research groups for active vibration control. Kumar et al. [4] and Umeda et al. [5] were among the pioneers to study the PZT generator and proposed an electrical equivalent model being converted from mechanical lumped models of a mass, a spring and a damper that describe a transformation of the mechanical impact energy into electrical energy in the PZT material. Kasyap et al. [6] formulated a lumped element model that represents the dynamic behavior of the PZT device in multiple energy domains and replaces them with electric circuit components. The model has been experimentally verified by using a one-dimensional beam structure. Huang et al. [7] and DeVoe et al. [8] did the displacement and tip-deflection analysis along the beam and made a comparison with the experimental results. However, both proposals were limited to the actuator mode. Williams et al. [9] analyzed a PZT structure by using a single degree of freedom mechanical model. However, the model did not extend to a bimorph multilayer structure. Lu et al. [10] improved the electrical model by adding an electromechanical coupling that represents a dynamic behavior of the beam vibrating under a single degree of freedom.

The fuzzy set theory was established by Zadeh, and it has been extensively researched in various fields of engineering. The main advantage of fuzzy logic controller (FLC) over conventional control approaches is that the FLC is considered artificial intelligence where control laws are designed by human intelligence based on expert's experience, not by a deterministic numerical calculation. FLC does not require the accurate mathematical model of the controlled object, and it can represent almost any deterministic controller. Therefore, FLC method has been applied widely for active vibration control of flexible structures Sharma et al. [11], Marinaki et al. [12] and Wei et al. [13]. In 1983, Brown and Yao [14] used the fuzzy theory to the engineering structures at first time. In 1986, Juang and Elton [15] adopted fuzzy logic to estimate the density of earthquake on the extent of damage for the constructions. Park [16] established the approximate model of the driver, the sensor and the fuzzy logic controller to solve the problems of vibrations for flexible structure. The result indicated that the fuzzy logic control had the stronger robust and self-adaptive for the linear and nonlinear system. Zeinoun and Khorrami [17] presented an adaptive control scheme based on fuzzy logic theory for active vibration control of smart beam structure. Experimental results were compared with the analytical results. Sharma et al. [11] used fuzzy logic controller to perform vibration control of cantilever beam using piezoelectric sensors and actuators and validated the same by performing experiments. Lin and Liu [18] experimentally controlled the active vibration of cantilever beam using simplified fuzzy logic controller and compared results with PD controller. Wei et al. [19] experimentally performed the active control of vibration of cantilever beam embedded collocated piezoelectric sensor/actuator pair using fuzzy logic controller. In this paper, smart cantilever beam is simplified into lumped parameter model, and fuzzy logic controller is used for vibration control.

2 Mathematical Modeling

Figure 1a shows a three-layered smart cantilever beam with two piezoelectric layers. The upper layer acts as sensor, and bottom layer acts as an actuator. The middle layer is substrate layer, made of stainless steel (SUS-304). Both sensor and actuator layer are made of PZT-PZNM ceramic. Figure 1b shows equivalent lumped parameter model of a smart cantilever beam to predict the vibratory response of free end, and Fig. 1c shows free body diagram of a lumped parameter model to derive the mathematical equation. The lumped parameter model simplifies the behavior of spatially distributed physical system into discrete entities that approximate the behavior of the distributed system under certain assumptions. Sensor, actuator and substrate material are parallel; therefore, equivalent stiffness of a smart cantilever beam can be calculated as:

$$k = K_S + K_a + K \tag{1}$$

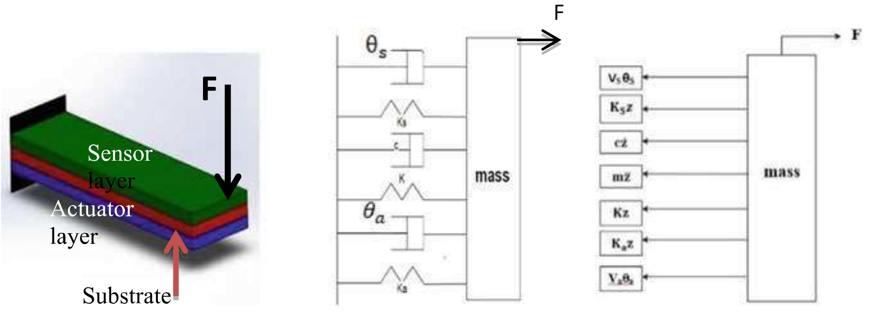
where K_{s}, K_{a} and K are the stiffness of sensor layer, actuator layer and substrate layer, θ_{s} and θ_{a} are the electrically induced damping coefficient of sensor and actuator layer, and c is the damping coefficient of smart cantilever beam.

Let $z(t)$ is the displacement; then, the equation of motion of the mechanical system is given by:

$$m\ddot{z}(t) + c\dot{z}(t) + kz(t) + \theta_{\mathrm{S}} V_{\mathrm{S}} + \theta_{\mathrm{a}} V_{\mathrm{a}} = F \tag{2}$$

where c is the damping coefficient, m is effective mass of smart cantilever beam, k equivalent stiffness of the smart cantilever beam, and F is the step force.

Equivalent stiffness of the smart cantilever beam is given by [20]:



(a) (b) (c)

Fig. 1 **a** Smart cantilever beam, **b** lumped parameter model of a smart cantilever beam and **c** free body diagram of a lumped parameter model

$$k = \frac{w}{4l^3}\left(\sum_{p=1}^{n_1} n_p E_p h_p^3 + \sum_{s=1}^{n_2} n_s E_s h_s^3\right) \tag{3}$$

where l is the length, w is the width of the beam, and n_p and n_s are number of layers in piezoelectric and substrate material. E_p and E_s are the Young's modulus of piezoelectric and substrate layer, respectively, and h_p and h_s are the respective heights of piezoelectric and substrate layer.

Equivalent mass of the smart cantilever beam is given by [20]:

$$m = 0.235wl\left(\sum_{p=1}^{n_1} n_p \rho_p h_p + \sum_{s=1}^{n_2} n_s \rho_s h_s\right) \tag{4}$$

$$w_n = \sqrt{\frac{k}{m}} \tag{5}$$

$$c = 2\xi w_n m \tag{6}$$

where ρ_p the density of piezoelectric layer, and ρ_s is density of substrate layer. w_n is natural frequency of a system, and ξ is damping factor.

The strain charge is given by [21]:

$$D_3 = d_{31}\sigma + \varepsilon_{33}^{s} E_3 \tag{7}$$

where is stress, d_{31} is the piezoelectric strain coefficient, E_3 is the electric field, D_3 is electric displacement, and ϵ_{33}^{s} is permittivity at constant strain.

The bending stress $\sigma(x)$ and moment $M(x)$ are given by:

$$\sigma(x) = \left(\frac{M(x)}{I}\right)h_{ps} \tag{8}$$

$$M(x) = \frac{kzl}{2} \tag{9}$$

$$E_3 = \frac{V}{h_p} \tag{10}$$

where V is voltage, and I is moment of inertia of the smart cantilever beam. Substituting the values of Eqs. (8), (9) and (10) in Eq. (7)

$$D_3 = \frac{d_{31}kzlh_{ps}}{2I} - \frac{\varepsilon_{33}^{s} V}{h_p} \tag{11}$$

The charge and current generated is given as:

$$q = D_3 bl = \frac{d_{31} kzlh_{ps}(bl)}{2I} - \frac{\varepsilon_{33}^{s} V(bl)}{h_p} \tag{12}$$

$$i = \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{V}{R} = \frac{d_{31} klh_{ps}(bl)}{2I}\left(\frac{\mathrm{d}z}{\mathrm{d}t}\right) - \frac{\varepsilon_{33}^{s}(bl)}{h_p}\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right) \tag{13}$$

$$\frac{V}{R} + C_P\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right) = \theta\left(\frac{\mathrm{d}z}{\mathrm{d}t}\right) \tag{14}$$

where $C_P = \frac{\varepsilon_{33}^{s}(bl)}{h_p}$ and $\theta = \frac{d_{31} klh_{ps}(bl)}{2I}$

$$q + C_P V = \theta z \tag{15}$$

Since there is no charge in the sensor layer, therefore $\{q = 0\}$ in Eq. (15). Sensor voltage and its rate of change with time are calculated as

$$V_S = \frac{\theta_S z}{C_p} \tag{16}$$

$$\frac{\mathrm{d}V_s}{\mathrm{d}t} = \frac{\theta_s}{C_p}\left(\frac{\mathrm{d}z}{\mathrm{d}t}\right) \tag{17}$$

When the beam is deformed, the sensor layer produces sensor voltage and rate of change of sensor voltage both the input is fed to the fuzzy logic controller. After applying control rules, the controller sends a control signal to the actuator layer in form of voltage called actuator voltage. A coupled equation of mechanical and electrical system is used to control the vibration of the smart cantilever beam.

$$m\ddot{z}_a + c\dot{z}_a + kz_a + \theta_a V_a = F - f_a \tag{18}$$

where V_a is the actuator voltage, θ_a is electrically induced damping coefficient, z_a is control deflection after applying the actuator voltage, and f_a is the actuator force will be determined using fuzzy logic controller.

3 Fuzzy Logic Controller Design

A controller is designed to regulate the undesired vibrations of the structure or system to the desired level by applying appropriate control actuation within shortest possible time. One of the most effective control algorithms used in vibration control is a fuzzy logic controller (FLC) which has a unique feature of converting the expert

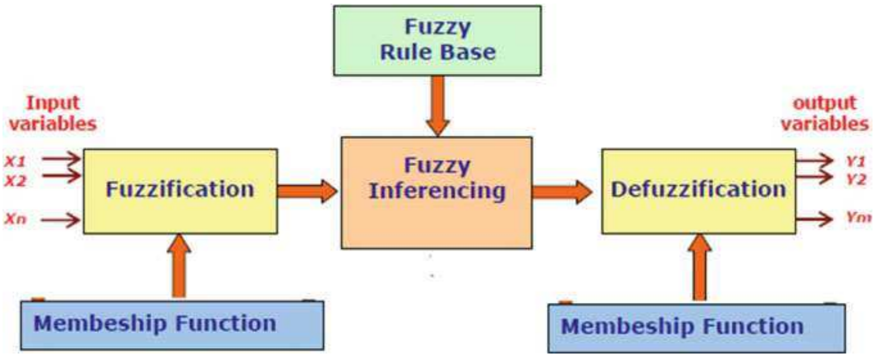


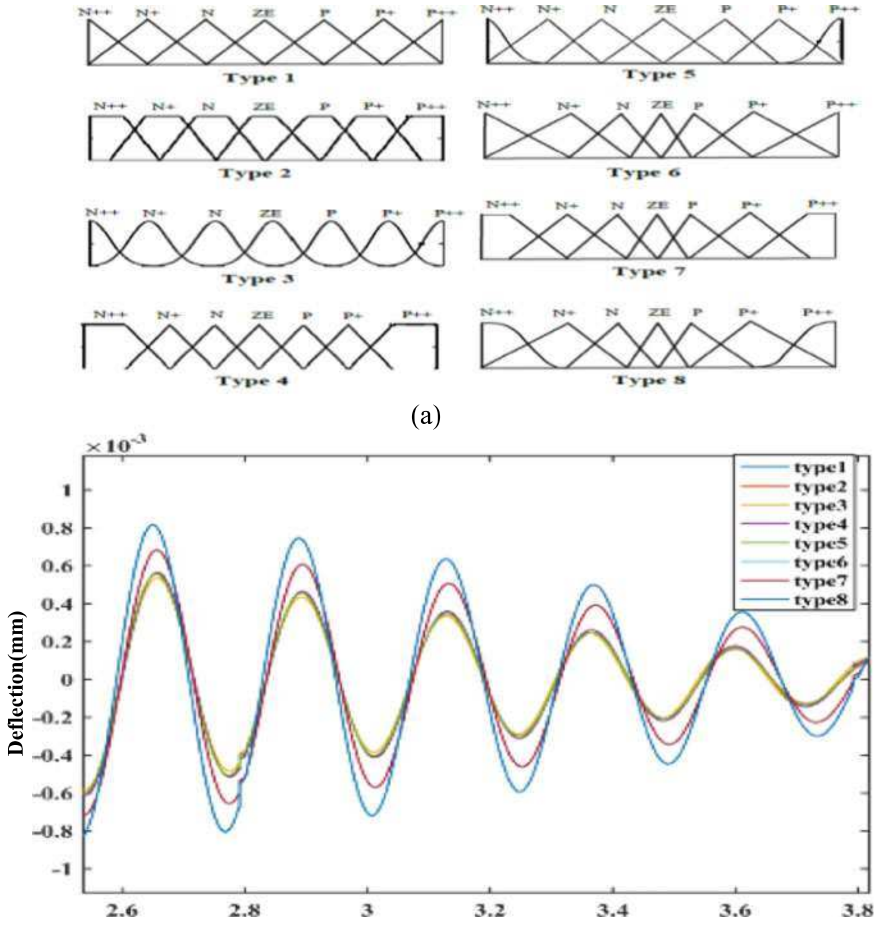
Fig. 2 Block diagram of fuzzy system

knowledge-based linguistic strategy into an automatic control strategy. FLC is based on converting the crisp inputs into fuzzy terms, using an expert knowledge base to find the control signal and finally converting this control signal into crisp output.

The fuzzy logic controller comprises of the following four elements (illustrated in Fig. 2) fuzzification, rule base, fuzzy inference and defuzzification. Input variables $X = [x_1, x_2, x_n]$ are crisp values, which are transformed into fuzzy sets in the fuzzification block. A fuzzification interface is used to convert the controller inputs into information that the inference mechanism can easily understand to stimulate and implement rules. A rule base contains a fuzzy logic quantification of the expert's linguistic description of how to achieve good control. Membership function provides a measure of the degree of elements in the universe of discourse U to fuzzy set. A defuzzification interface converts the conclusions of the inference mechanism into actual inputs for the process.

The present controller is designed as double-input and single-output (DISO) fuzzy logic system. Therefore, the sensor voltage which is obtained from piezoelectric sensor subjected to structural deflection is used as one input variable while the rate of change of the sensor voltage is used as second input to the fuzzy logic controller. The voltage signal is to be provided to piezoelectric actuator which has been chosen as output variable for designed fuzzy logic controller. The actuator generates control force on the structure to counter the structural vibrations.

In order to select the shapes of different membership function, the tuning of the membership functions is carried out. Figure 3a shows eight different types of membership function are selected covering almost all the combinations. The active vibration control analysis is carried out on smart cantilever beam to select best combination of membership functions as shown in Fig. 3b. The geometric properties, material properties, loading conditions and boundary conditions are kept same for all the numerical simulations. Figure 4 presents the active vibration control of smart cantilever beam using different combinations of membership functions. The combination of membership functions of type-3 demonstrates the best performance among the cases under study.



(a)

(b)

Fig. 3 **a** Different combinations for fuzzy membership functions. **b** Performance evaluation of different combinations of membership functions

Fuzzy Rule Base Set of 49 rules is used in present fuzzy logic controller. To define these membership functions and fuzzy rules input and output variables are described by seven linguistic terms, positive large (P++), positive medium (P+), positive small (P), zero (ZE), negative small (N), negative medium (N) and negative large (N++) (Table 1).

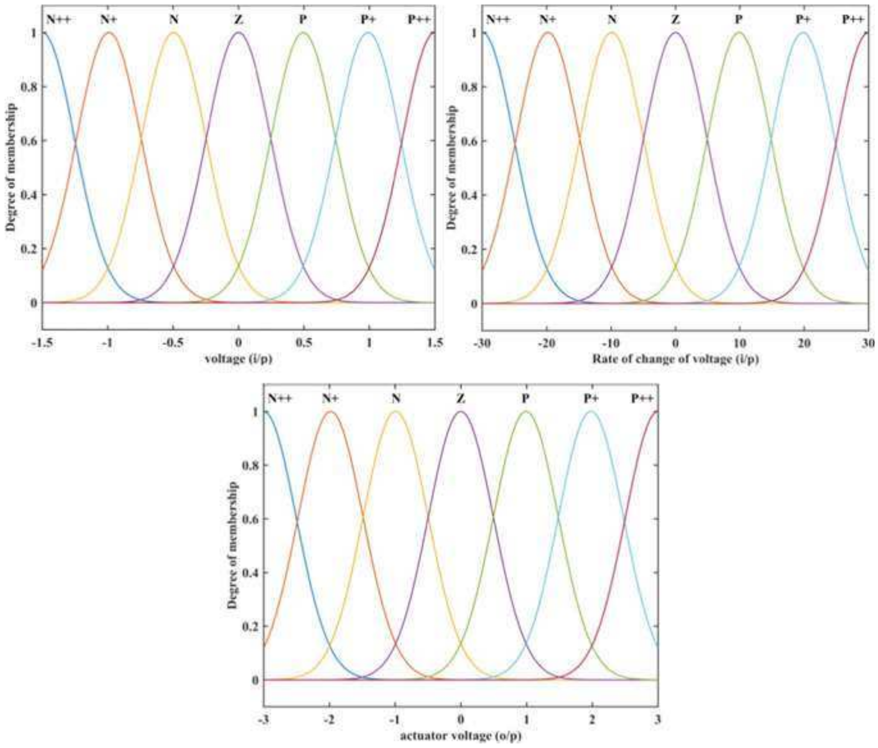


Fig. 4 Fuzzy membership functions for input (sensor voltage and rate of change of sensor voltage) and output (actuator voltage) variables

Table 1 Fuzzy rule base

$V_S/\dot{V}_s$	N++	N+	N	ZE	P	P+	P++
N++	P++	P++	P++	P++	P+	N	N
N+	P++	P++	P+	P++	P	ZE	N
N	P++	P+	P+	P+	ZE	N+	N+
ZE	P++	P+	ZE	ZE	ZE	N+	N++
P	P+	P+	ZE	N+	N+	N+	N++
P+	N	ZE	N	N++	N	N++	N++
P++	N	N	N+	N++	N+	N++	N++

4 Result and Discussion

Material and geometric properties of the smart cantilever beam are given in Table 2. Equations (2), (16) and (17) are used to develop a Simulink model as shown in Fig. 5 and used to calculate the value of deflection, sensor voltage and rate of change

Table 2 Material and geometric properties

Parameter	Numerical value
Dimensions of substrate material	400 × 25 × 1 mm^3
Dimensions of piezoelectric material	400 × 25 × 1 mm^3
Density of piezoelectric material	7600 kg/m^3
Density of substrate material	8000 kg/m^3
Young modulus of piezoelectric material	72.46 GPa
Young modulus of substrate material	193 GPa
Relative permittivity ($\varepsilon_{33}/\varepsilon_0$)	2300
Piezoelectric coupling constant (d_{31})	-200×10^{-12} C/N

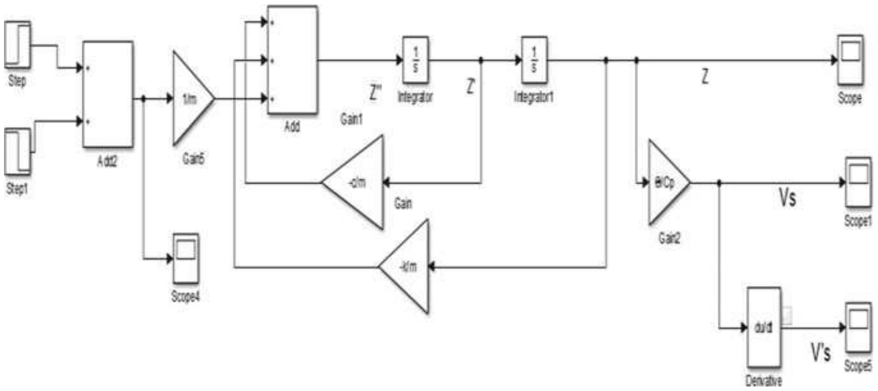


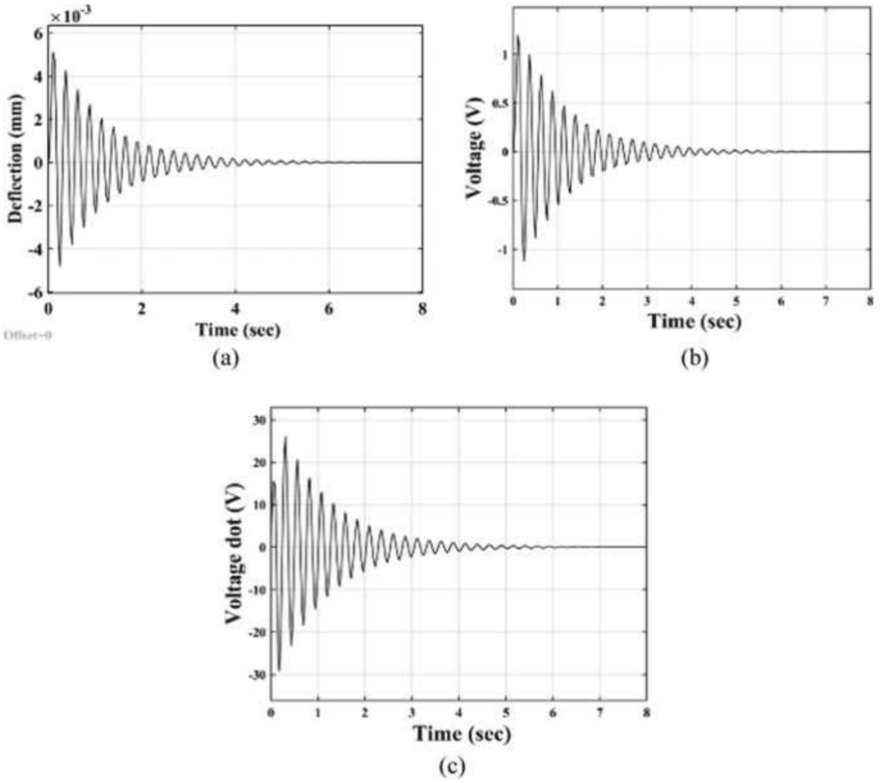
Fig. 5 Simulink model of smart cantilever beam to calculate deflection, sensor voltage and rate of change of sensor voltage

of sensor voltage. In this Simulink model, step force of 0.1 N is applied for 0.1 s on smart cantilever beam because of this force a structural deflection of ±5 mm, sensor voltage ±1.2 V, rate of change of sensor voltage ± 30 V/s is calculated.

Material and geometric properties used are shown in Table 2 [20]:

The simulation output of the smart cantilever beam is shown below in Fig. 6 where Fig. 6a shows deflection vs. time response, Fig. 6b shows voltage vs. time response, and Fig. 6c shows rate of change of sensor voltage vs. time response.

The value of sensor voltage and rate of change of sensor voltage acts as an input for the fuzzy logic controller to calculate actuator voltage. The value of actuator voltage is substituted in Eq. (18) to control the vibration of smart structure. Figure 7 shows Simulink model of smart cantilever beam using fuzzy logic controller. Figure 8 shows comparisons between controlled and uncontrolled response with respect to time. Further study will be performed to compare the controlled response from different type of controller.



(a) (b)

(c)

Fig. 6 Simulation output of smart cantilever beam

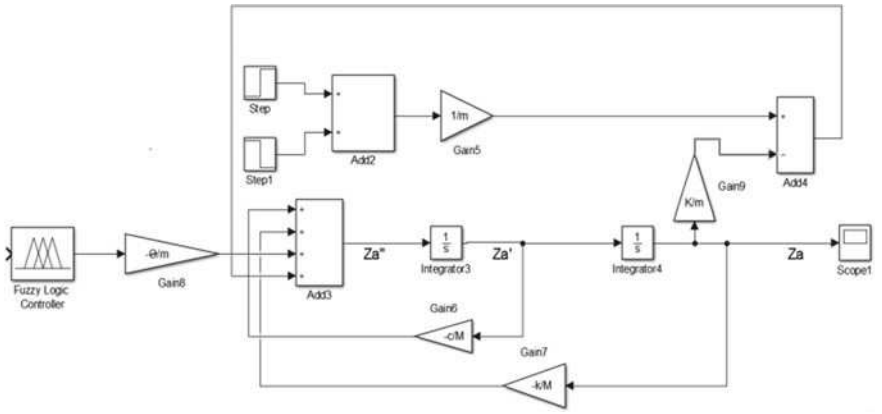


Fig. 7 Simulink model of smart cantilever beam using fuzzy logic controller

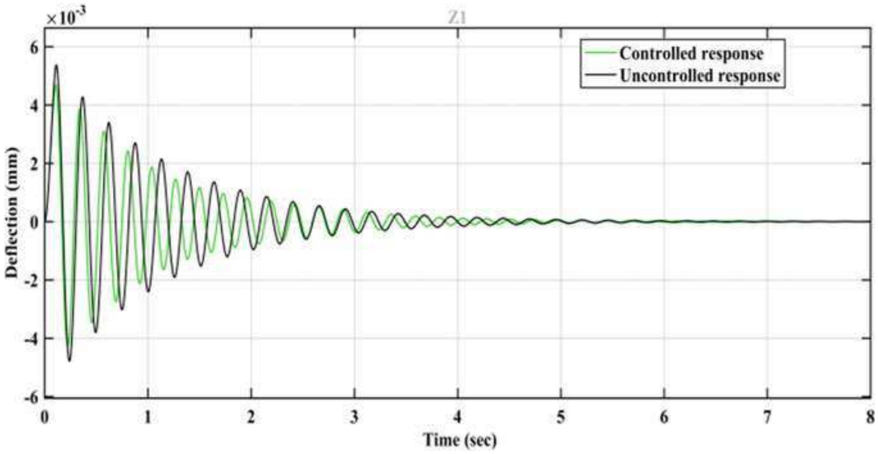


Fig. 8 Simulink output of smart cantilever beam using fuzzy logic controller

5 Simulation Output

See Fig. 8.

6 Conclusion

In this paper, smart cantilever beam is simplified into lumped parameter model, and fuzzy logic controller is used for vibration control. The sensor voltage and its derivative are given as inputs to the controller which computes actuator voltage using 49 if–then type rules. It has been find out that combination of Gaussian membership function at ±3 V of actuator voltage give, the best controlled response. This type of simplified model would be a very powerful tool to guide the design of the device structure and can be applied in real environment.

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